

**KEY** 4-11-16 & 4-13-16

**Algebra Practice AIR**

work days in class

Show work for problems marked with ★

**I. NON-CALCULATOR SECTION**

1★ Which expression is equivalent to  $(3x^5 + 8x^3) - (7x^2 - 6x^3)$ ?

- A  $-4x^3 + 14$
- B  $-4x^5 + 14x^3$
- C  $3x^5 + 14x^3 - 7x^2$
- D  $3x^5 + 2x^3 - 7x^2$

$$3x^5 + 8x^3 - 7x^2 + 6x^3$$

$$3x^5 + 14x^3 - 7x^2$$

2★ Which points are on the graph of the equation  $-3x + 6y + 5 = -7$ ?

Select all that apply.

- A  $(-3, 6)$
- B  $(-2, 0)$
- C  $(0, -2)$
- D  $(6, -3)$
- E  $(8, 2)$

$$-3(-3) + 6(6) + 5 = -7$$

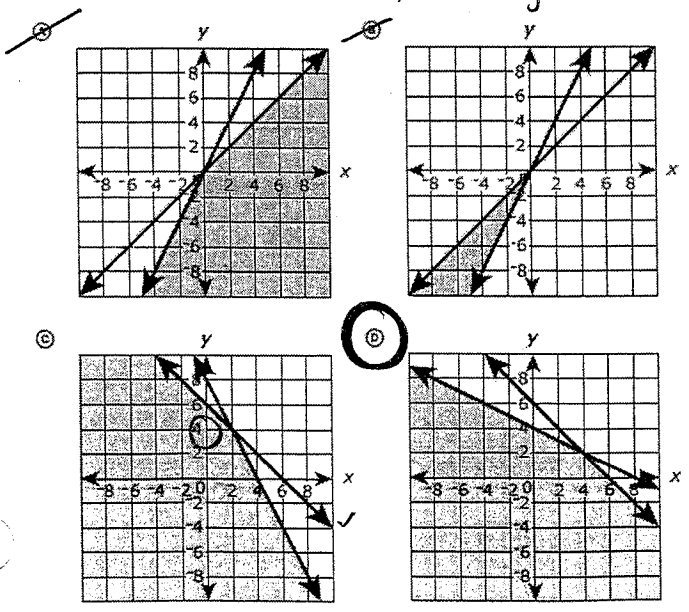
$$-3(-2) + 6(0) + 5 = -7$$

3. Which graph best represents the solution to this system of inequalities?

$$x + y \leq 6 \rightarrow y \leq 6 - x$$

$$x + 2y \leq 8 \rightarrow y \leq 4 - \frac{1}{2}x$$

Neg. Slopes



4★ Which factorization can be used to reveal the zeros of the function  $f(n) = -12n^2 - 11n + 15$ ?

- A  $f(n) = -n(12n + 11) + 15$
- B  $f(n) = (-4n + 3)(3n + 5)$
- C  $f(n) = -(4n + 3)(3n + 5)$
- D  $f(n) = (4n + 3)(-3n + 5)$

$$-12n^2 - 11n + 15$$

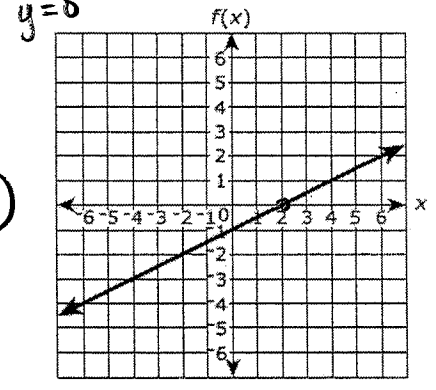
$$-(12n^2 + 11n - 15)$$

$$-(\frac{12n+20}{4})(\frac{12n-9}{3})$$

$$-(3n+5)(4n-3)$$

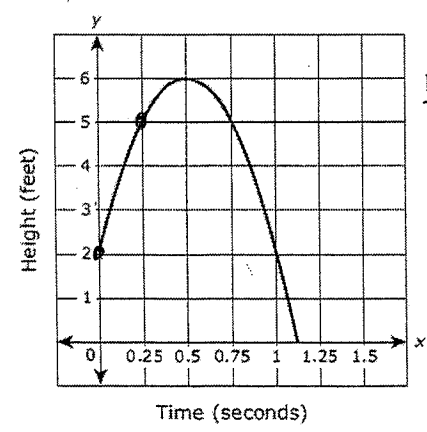
\* Have to play around w/ negative.

5. The graph of the function  $f(x) = -1 + 0.5x$  is shown on the coordinate plane. For what value of  $x$  does  $f(x) = 0$ ?



$x = 2$

6★ A tennis ball was 2 feet off the ground when a tennis player hit it so that the ball traveled up in the air before coming back to the ground. The height of the tennis ball is described by the graph shown. Numbers along the x-axis represent the time, in seconds, after the ball was hit, and the numbers along the y-axis represent the height, in feet, of the ball at time  $x$ .



$$\frac{\text{Rise}}{\text{Run}} = \frac{3}{0.25}$$

$$= 12$$

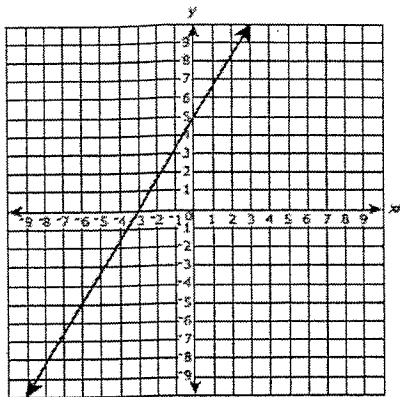
Use the graph to estimate the average rate of change of the height of the ball for the first 0.25 second after being hit.

- A 0.75 feet per second
- B 3.0 feet per second
- C 12 feet per second
- D 20 feet per second

180  
1:180  
2:90  
3:60  
4:45  
5:30  
6:30  
9:20

7. Which graph represents the equation  $5y - 3x = -15$ ?

(A)

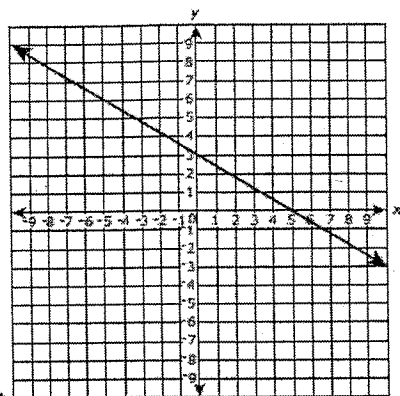


$$5y = -15 + 3x$$

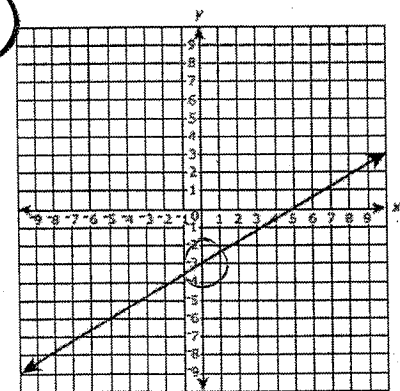
$$y = -3 + \frac{3}{5}x$$

↑  
positive  
m

~~(B)~~

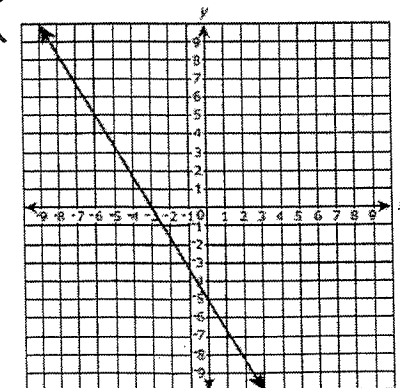


(C)



← Correct y-int.

~~(D)~~



## II. CALCULATOR SECTION

8. The formula for finding the perimeter,  $P$ , of a rectangle with length  $l$  and width  $w$  is given.

$$P = 2l + 2w$$

Which formula shows how the length of a rectangle can be determined from the perimeter and the width?

(A)  $l = \frac{P}{2} - 2w$

(B)  $l = \frac{P-2w}{2}$

(C)  $l = \frac{P}{2} + w$

(D)  $l = \frac{P-2}{2w}$

$$P = 2l + 2w$$

$$\quad \quad \quad -2w \quad \quad -2w$$

$$\frac{P-2w}{2} = \frac{2l}{2}$$

$$\frac{P}{2} - w = l$$

$$\frac{P-2w}{2} = l$$

9. At the beginning of an experiment, the number of bacteria in a colony was counted at time  $t = 0$ . The number of bacteria in the colony  $t$  minutes after the initial count is modeled by the function  $b(t) = 4(2)^t$ . Which value and unit represent the average rate of change in the number of bacteria for the first 5 minutes of the experiment?

Select all that apply.

(A) 24.0

(B) 24.8

(C) 25.4

(D) 25.6

(E) bacteria

(F) minutes

(G) bacteria per minute

(H) minutes per bacteria

$$b(t) = 4(2)^t$$

$$b(t) = 4(2)^5$$

$$= 128 \text{ bacteria after 5 min.}$$

$$\frac{128 \text{ bacteria}}{5 \text{ min}} = 25.6 \text{ bacteria/min}$$

Use the information provided to answer Part A through Part C for question 10.

Consider the three points  $(-4, -3)$ ,  $(20, 15)$ , and  $(48, 36)$ .

10. Part A

Which points are on the same line that passes through  $(-4, -3)$ ,  $(20, 15)$ , and  $(48, 36)$ ?

Select all that apply.

- (a)  $(-8, -6)$
- (b)  $(-2, -1)$
- (c)  $(0, 0)$
- (d)  $(4, 3)$
- (e)  $(6, 8)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{15 + 3}{20 + 4} = \frac{18}{24} = 0.75$$

$$y + 3 = 0.75(x + 4)$$

$$y + 3 = 0.75x + 3$$

$$y = 0.75x$$

Part B

Use the information from Part A to explain why the ratio of the  $y$ -coordinate to the  $x$ -coordinate is the same for any point on the line except the  $y$ -intercept.

Explain why this is not true for the  $y$ -intercept.

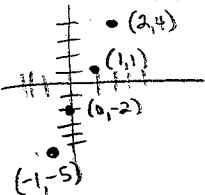
Enter your explanations in the space provided.

$y$ -int is when  $x=0$ ...  
 so whenever you set up  $\frac{y}{x}$ , you'd have  $\frac{\#}{0} = \text{undefined}$   
 THOU SHALT NOT DIVIDE BY ZERO!

Part C

Do the points on the line  $y = 3x - 2$  have a constant ratio of the  $y$ -coordinate to the  $x$ -coordinate for any point on the line except the  $y$ -intercept? Explain your answer.

Enter your answer and your explanation in the space provided.



$\frac{y}{x} = \frac{4}{2} = \frac{1}{1} = \frac{-5}{-1}$  No, the line does not pass through the origin - only those that do have a constant ratio of  $y$  to  $x$ .

**I CORRECTED this problem for the "20% remained" on the last page.**  
 Use the information provided to answer Part A through Part C for question 11.

Phil and Matt made cookies for a fundraiser at their high school.

- Phil made 25% more cookies than Matt.
- The cookies sold for \$0.25 each.
- After the sale, 20% of the combined total of their cookies remained.

11. Part A

Create an equation to represent the total amount of money Matt and Phil earned at the fundraiser based on the number of cookies Matt made. Explain how you determined your equation.

Enter your equation and your explanation in the space provided.

$M = \text{amt Matt made}$   
 $P = \text{Phil's cookies } (= m + 0.25m)$

$$T = M + P$$

$$T = \underline{m} + \underline{m} + 0.25m$$

$$T = (2.25m) 0.25$$

$$T = 0.5625m$$

- Added Matt & Phil's cookies, but Phil = Matt + 25% Matt  
 - multiplied that by 25¢.

Part B

Phil and Matt made a total of \$72.00 selling the cookies. How many cookies did Phil make and how many cookies did Matt make? Show your work.

Enter your answers and your work in the space provided.

~~$72.00 = 0.5625m$   
 $128 = m$   
 Matt = 128 cookies  
 Phil =  $128 + 0.25(128) = 160$  cookies~~

Part C

Next year Phil and Matt may sell the cookies for \$.50 each. They plan to make the same total number of cookies, but they predict that they will only sell 70% of them given the price increase. Based on their prediction, should Phil and Matt raise the price of the cookies? Justify your answer.

Enter your answer and your justification in the space provided.

~~Sold 208 total cookies  
 70% = 201.6 cookies  
 $201.6(0.50) = \$100.80$~~   
 Yes they will still make \$28.80 more than last year.

12. Let  $|x| + |y| = c$ , where  $c$  is a real number.

Determine the number of points that would be on the graph of the equation for each given case:

Case 1:  $c < 0$

Case 2:  $c = 0$

Case 3:  $c > 0$

Justify your answers.

Enter your answers and justifications in the space provided.

Case 1: (when values are negative)

\* no points on the line (you can't add 2 positives and get a negative)

Case 2: (when  $c = 0$ )

\*  $|0| + |0| = 0$  ← only happens one way  
→ one point

Case 3: (when  $c$  is positive)

\* Infinitely many points (there are TONS of ways a (+) plus (+) gives you a (+).)

13. What is one solution of the equation  $x^2 - 21.75x = -15.75$ ?

Enter your answer in the box.

$$x^2 - 21.75x = -15.75$$

$$+15.75 \quad +15.75$$

$$x^2 - 21.75x + 15.75 = 0$$

$$x = \frac{21.75 \pm \sqrt{(21.75)^2 - 4(1)(15.75)}}{2(1)}$$

$$x = \frac{21.75 \pm \sqrt{410.06}}{2}$$

$$x = 0.75 \quad \text{OR} \quad x = 20.99$$

14. If  $a$  is a non-zero, real number and  $a(x-3)^2 - b = c$ ,

• Prove that  $x = 3 \pm \sqrt{\frac{b+c}{a}}$ . Show your work.

• If  $a = 2$  and  $b = 5$ , determine what condition(s) on  $c$  will restrict solutions for  $x$  to real numbers.

Explain your reasoning.

Enter your proof, your answer, and your explanation in the space provided.

$$a(x-3)^2 - b = c$$

$$+b \quad +b$$

$$\frac{a(x-3)^2}{a} = \frac{b+c}{a}$$

$$\sqrt{(x-3)^2} = \sqrt{\frac{b+c}{a}}$$

$$x-3 = \pm \sqrt{\frac{b+c}{a}}$$

$$+3 \quad +3$$

$$x = 3 \pm \sqrt{\frac{b+c}{a}}$$

$$a=2, b=5$$

$$x = 3 \pm \sqrt{\frac{5+c}{2}}$$

↳  $c$  has to be -5 OR more (so you don't get a neg. # under the  $\sqrt{\quad}$ )

15. Consider the following claim: If the point  $(2+d, y)$  is on the graph of the function  $f(x) = x(x-4)$ , then the point  $(2-d, y)$  is also on the graph.

• Use algebra to show that the claim is true.

• What is the relationship between the line  $x = 2$  and the graph of  $f(x)$ ? Justify your reasoning.

Enter your work, your answer, and your justification in the space provided.

$$f(x) = x(x-4)$$

$$y = (2+d)(2+d-4)$$

$$y = (2+d)(d-2)$$

$$y = 2d-4+d^2-2d$$

$$y = d^2-4$$

$$y = (2-d)(2-d-4)$$

$$y = (2-d)(-d-2)$$

$$y = -2d-4+d^2+2d$$

$$y = d^2-4$$

SAME!

$$f(x) = x(x-4) \rightarrow \text{Vertex: } \frac{-b}{2a} = \frac{4}{2(1)} = \frac{4}{2} = 2$$

$$f(x) = x^2 - 4x \rightarrow (2, -4)$$



x	y
0	0
1	-3
2	-4
3	-3
4	0

$x = 2$  is the axis of symmetry of the graph of  $f(x) = x(x-4)$ .

Use the information provided to answer Part A and Part B for question 16.

The Water Watch program is encouraging customers to reduce the amount of water they use each day. The program is selling low-flow showerheads, which use 2 gallons of water per minute, for \$54.00 each.

A family currently has a showerhead that uses 5 gallons of water per minute and is considering replacing it with one of the low-flow showerheads. The family uses the shower an average of 20 minutes per day and pays \$0.002 per gallon of water.

16. Part A

Create a model that can be used to determine the cost savings, in dollars, for the family to purchase and use a low-flow showerhead in terms of the number of days.

Then determine the number of days at which the family will start saving money. Justify your answer in terms of the context.

Enter your model, answer, and justification in the space provided.

Now: 5 gal/min  
20 min/day → 100 gal/day → \$0.20/day  
\$0.002/gal

New: 2 gal/min  
20 min/day → 40 gal/day → \$0.08/day  
\$0.002/gal

Save \$0.12/day

	Now	New	
	$0.20x = 54 + 0.08x$		
	$-0.12x$	$-0.12x$	
	$f(x) = 54 - 0.12x$		* model *

Start saving \$ at day 451

0.12x = 54 + 0.08x  
-0.08x  
0.04x = 54  
x = 450 (when 2 plans are equal)

Part B

One year after the low-flow showerhead is purchased, the cost of water increases by 5%. Create a new model to determine the cost savings, in dollars, with the increase in the cost of water.

Use your model to determine the number of days at which the family will start saving money after the increase in the cost of water. Justify your answer.

Enter your model, answer, and justification in the space provided.

5% increase =  $0.12 + 0.05(0.12) = 0.126$  (NEW)

Plug 365 into the model from part A

$f(x) = 54 - 0.12(365)$   
= \$10.2 left to pay

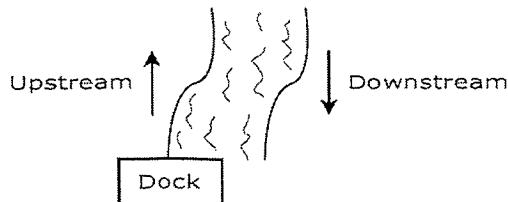
So after 1 year, the model is

$f(x) = 10.2 - 0.126x$  \* model

$0 = 10.2 - 0.126x$   
 $-10.2 = -0.126x$   
 $80.95 = x$

This is when they will break even → they will start saving \$ on day 81

17. Gabriel operates a riverboat and frequently offers tours of the river. Typically a tour lasts for 3.25 hours. The riverboat usually takes 2.00 hours to make the 25-mile trip upstream from the dock and 1.25 hours to make the 25-mile return trip downstream.



Gabriel is considering offering a shorter tour that will last 2.50 hours and travel only 20 miles upstream before returning. Will the shorter tour be possible if the riverboat travels at the same speed as it does in the 3.25-hour tour? Show your steps and justify your answer.

Enter your answer, your work, and your justification in the space provided.

↑ = 2.00 hrs = 25 miles  
↓ = 1.25 hrs = 25 miles

$\frac{25}{2} = 12.5 \text{ mph } \uparrow$        $\frac{25}{1.25} = 20 \text{ mph } \downarrow$

↑ = 20 miles @ 12.5 mph = 1.6 hrs.  
↓ = 20 miles @ 20 mph = 1 hr.

No - the shorter tour is not possible in 2.5 hrs... it would take 2.6 hrs. to make it possible.

This will let you know how much of shower head they still have to pay

Use the information provided to answer Part A and Part B for question 18.

A high school is having a talent contest and will give different prizes for the best 5 acts in the show. First place wins the most money, and each place after that wins \$50 less than the previous place.

18. Part A

Create a model that can be used to determine the total amount of prize money based on the value of the first place prize.

Enter your model in the space provided.

$$1^{\text{st}} \text{ place} = X$$

$$2^{\text{nd}} = X - 50$$

$$3^{\text{rd}} = X - 100$$

$$4^{\text{th}} = X - 150$$

$$5^{\text{th}} = X - 200$$

$$T = X + X - 50 + X - 100 + X - 150 + X - 200$$

$$T = 5X - 500$$

Part B

The talent contest has a total of \$1,000 in prize money. What is the amount of money for each of the five prizes? Show your work.

Enter your answers and your work in the space provided.

$$1000 = 5X - 500$$

$$1500 = 5X$$

$$300 = X$$

$$1^{\text{st}} = \$300$$

$$2^{\text{nd}} = \$250$$

$$3^{\text{rd}} = \$200$$

$$4^{\text{th}} = \$150$$

$$5^{\text{th}} = \$100$$

#11

 $m = \# \text{ cookies Matt made}$  $T = \text{total } \$$ 

Part A → Since 20% of their cookies remained, that means they sold 80%

$$0.80(0.5625)m = T$$

$$0.45m = T$$

Part B →  $\frac{0.45m}{0.45} = \frac{72}{0.45}$

$$m = 160 \text{ cookies Matt made}$$

$$\text{Phil} = m + 0.25m = 160 + 0.25(160) = 200 \text{ cookies Phil made}$$

Part C → They sold 360 total cookies

70% of that amount =  $0.70(360) = 252$  cookies will sell next year

If they sell them at 50¢ each

$$(0.50)(252) = \$126.00$$

they can expect to make this amount.

Should they raise the price? Explain.

YES - they will still make \$54.00 more than this year.

next year  
↓  
126 - 72 = 54  
this year